It takes a patient at most 30 minutes to respond to an injection of a certain drug. A randomly selected patient is given an injection of the drug, and X is the random variable representing the time it takes for patient to respond (measured in units of hours). **SCORE: _____ / 10 PTS**

If the probability density function is given by $f(x)=\begin{cases}\frac{k}{\sqrt{1-x^2}}, & x\in[0,\frac{1}{2}]\\ 0, & x\notin[0,\frac{1}{2}]\end{cases}$ for some constant k, find the mean (average) response time.

$$\int_0^{\frac{1}{2}} \frac{k}{\sqrt{1-x^2}}\,dx = 1$$

$$k \sin^{-1} x \Big|_0^{\frac{1}{2}} = 1$$

$$k \cdot \frac{\pi}{6} = 1$$

$$k = \frac{6}{\pi}$$

$$\frac{6}{\pi}\int_0^{\frac{1}{2}} \frac{x}{\sqrt{1-x^2}}\,dx = \frac{6}{\pi}\cdot -\frac{1}{2}\int_1^{\frac{3}{4}} u^{-\frac{1}{2}}\,du = -\frac{3}{\pi}\left(2u^{\frac{1}{2}}\right)\Big|_1^{\frac{3}{4}}$$

$u = 1-x^2$ $\quad x=\frac{1}{2} \longrightarrow u=\frac{3}{4}$, $\quad x=0 \longrightarrow u=1$

$du = -2x\,dx$

$-\frac{1}{2}\,du = x\,dx$

(2)

$$= -\frac{6}{\pi}\left(\frac{\sqrt{3}}{2}-1\right)$$

$$= \frac{6}{\pi}\left(1-\frac{\sqrt{3}}{2}\right)$$

HOURS

ALL ITEMS (1) POINT

UNLESS OTHERWISE NOTED

Find the length of the curve $y = \dfrac{2+x^6}{8x^2}$ from the point $(1, \frac{3}{8})$ to the point $(2, \frac{33}{16})$.

SCORE: _____ / 6 PTS

$y = \frac{1}{4}x^{-2} + \frac{1}{8}x^4$

$y' = \underbrace{-\frac{1}{2}x^{-3} + \frac{1}{2}x^3}$

$\underbrace{\int_1^2 \sqrt{1 + \left(-\frac{1}{2}x^{-3} + \frac{1}{2}x^3\right)^2}\,dx}$

$= \int_1^2 \sqrt{1 + \frac{1}{4}x^{-6} - \frac{1}{2} + \frac{1}{4}x^6}\,dx$

$= \int_1^2 \sqrt{\underbrace{\frac{1}{4}x^{-6} + \frac{1}{2} + \frac{1}{4}x^6}}\,dx$

$= \underbrace{\int_1^2 \left(\frac{1}{2}x^{-3} + \frac{1}{2}x^3\right)dx}$

$= \underbrace{\left(-\frac{1}{4}x^{-2} + \frac{1}{8}x^4\right)\Big|_1^2}$

$= \left(-\frac{1}{16} + 2\right) - \left(-\frac{1}{4} + \frac{1}{8}\right)$

$= \frac{31}{16} + \frac{1}{8}$

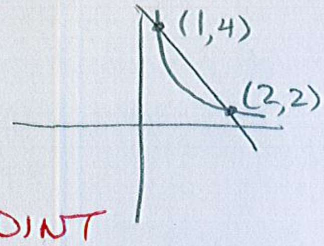
$= \underbrace{\frac{33}{16}}$

ALL ITEMS

1 POINT

Find the center of mass of the region between the curves $y = \frac{4}{x}$ and $y = 6 - 2x$.

SCORE: _____ / 14 PTS



$$\frac{4}{x} = 6 - 2x$$

$$4 = 6x - 2x^2$$

$$2x^2 - 6x + 4 = 0$$

$$2(x-1)(x-2) = 0$$

$$x = 1, 2$$

ALL ITEMS 1 POINT
UNLESS OTHERWISE NOTED

$$A = \int_1^2 \left(6 - 2x - \frac{4}{x}\right) dx = \left(6x - x^2 - 4\ln|x|\right)\Big|_1^2 = (12 - 4 - 4\ln 2) - (6 - 1)$$

$$= 3 - 4\ln 2$$

$$\int_1^2 x\left(6 - 2x - \frac{4}{x}\right) dx = \int_1^2 (6x - 2x^2 - 4)\, dx = \left(3x^2 - \frac{2}{3}x^3 - 4x\right)\Big|_1^2$$

$$= \left(12 - \frac{16}{3} - 8\right) - \left(3 - \frac{2}{3} - 4\right)$$

$$= \frac{1}{3}$$

$$\frac{1}{2}\int_1^2 \left((6-2x)^2 - \left(\frac{4}{x}\right)^2\right) dx = \frac{1}{2}\int_1^2 \left(36 - 24x + 4x^2 - \frac{16}{x^2}\right) dx$$

$$= \int_1^2 \left(18 - 12x + 2x^2 - \frac{8}{x^2}\right) dx$$

$$= \left(18x - 6x^2 + \frac{2}{3}x^3 + \frac{8}{x}\right)\Big|_1^2$$

$$= \left(36 - 24 + \frac{16}{3} + 4\right) - \left(18 - 6 + \frac{2}{3} + 8\right)$$

$$= \frac{2}{3}$$

$$\text{CENTER OF MASS} = \frac{1}{3 - 4\ln 2}\left(\frac{1}{3}, \frac{2}{3}\right)$$

$$= \left(\frac{1}{9 - 12\ln 2}, \frac{2}{9 - 12\ln 2}\right) \quad 2$$